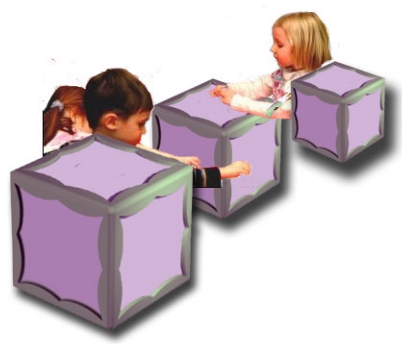


Early Childhood Numeracy

Numeracy refers to the understanding and ability to reason with number. It involves our overall conception of number and our use of number concepts to perform operations. Number operations can range from simple addition to complex algebraic applications, and number concept will influence every sphere of our mathematical learning for the remainder of our lives. Though there are clearly limits to preschool children's capacity to reason numerically at certain levels, there are many fundamental mathematical concepts that are fully within their grasp. These concepts, when taken together, provide a sound foundation for further conceptual understanding. They also prepare the young child for the coming challenges of the mathematics curriculum in kindergarten through grade 2—a goal supported by the U.S. national *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000). As with older students at any level (National Research Council [NRC], 1999a, 1999b), the goals and challenges of preschool mathematics teaching and learning are largely determined by children's prior conceptions. However, the situation is somewhat unique at the preschool level—for very young children, these prior conceptions have been primarily acquired in the absence of intentional or formal instruction. Therefore, it is necessary to identify and structure initial activities according to what children have acquired naturally through interaction with their environment, and provide well-planned opportunities for children to build upon their informally acquired knowledge and abilities, or even to help them form their initial constructions at the earliest ages.

Early number concept: Finding the springboard for further learning

> Implicit conceptions of number develop very early in young children as a consequence of environment.

From birth children are exposed to fundamental informal mathematics through interaction with their surrounding social environment. As a natural consequence, it is not unusual for very young children to develop basic implicit notions of such concepts as number (i.e., more or less) and space or location (i.e., here or there). Regardless of culture, children's exposure to rich counting systems is virtually assured (Lave, 1988; Rogoff, 1990). Through interaction, most children will learn the counting words. Though these understandings are largely language-embedded, they nonetheless begin to expose children to certain principles that are essential precursors to further formal or operational understanding of mathematics concepts (Gelman & Gallistel, 1978). This exposure varies, however, as does the degree to which children progress. Study findings suggest wide variance in the level of natural development of intuitive number understanding children attain during their early preschool years (Case, 1985; Case, Griffin & Kelly, 1999; Griffin & Case, 1996, 1998; Griffin, Case, & Capodilupo, 1995; Griffin, Case, & Siegler, 1994; Hiebert, 1986; Siegler & Robinson, 1982). Still, analyses of findings reveal several general characteristics of preschool learning experiences that tend to produce the children who are most prepared for the elementary curriculum to come. In general, experiences that are most successful for helping preschool children form strong initial concepts of number and operations are:

Optimal experiences for learning number and operations at the preschool level are:

- developmentally appropriate;

- driven by language and process; and

- thoughtfully sequenced and incremental.

Developmentally appropriate. Activities that are appropriate for young children have more to do with the *way* that new understandings are approached and the *level* to which they are addressed than with the understandings themselves. There are many considerations, but three in particular stand out—using the children themselves as referents, restricting the number of concepts children must think about at one time, and maintaining an attitude of play.

Using the **child as a referent** is not only in keeping with what we know about children's egocentric nature, but is also particularly useful in working with small numbers (e.g., counting on fingers and toes) and initial concepts of length and distance (e.g., children's height, foot length). Though not new knowledge—it goes back to Piaget's concept of egocentrism in young children—it is seldom put to work instructionally. Its importance has been borne out in research many times, and methods that use children as referents have not only consistently been found to be effective, but children actually seem bored when many concepts are approached in any other manner (Lay & Dopyera, 1977). When we **restrict concepts** that must be considered in tandem, children are more able to form early conceptions of number. For example, young children just learning their counting words will usually comprehend their task and remember the words better if they are not expected to simultaneously order the objects they are counting. In fact, we may wish to initially provide children only with objects that are identical. Many would claim that isolating learning objectives reduces learning to an artificial level, but this is nonsensical. We are simply focusing learning as it best suits the individual child—there is always room later to raise the bar through incorporating various additional concepts. When we maintain an **attitude of play**, we are taking into account a wealth of research regarding how very young children best learn. Read our article Play With Me! to find out more about using play to promote early learning.

Language- and process-driven. Children's early learning experiences are very dependent on language. Counting words and mathematical vocabulary play a very important role (see our article Count With Me! and the Research Précis - Early Numeracy: Number and Language). Likewise, children's use of continually more complex procedures has been found to be the most consistent factor in advancing early understanding and ability to perform operations, even simple operations such as addition of whole numbers (see our article Add With Me!).

Sequenced instructional designs have two major effects on a young child's learning. First, by paying careful attention to what a child understands and the incremental nature of objectives to come, we ensure that the two previous characteristics of successful learning experiences are more fully satisfied. Second, we recognize and adjust the way in which we provide experiences to young children to accommodate future requirements. Every concept that a child learns is part of an ongoing continuum of mathematical concepts that lead into the kindergarten and early elementary mathematics curricula and required standards for learning. In this sense, the necessary learning that takes place in preschool is the beginning stage of a chain of understanding that develops over a period of years—a *trajectory of number learning*.

The first two have to do with style—they are overarching considerations for how we put our plans into action for promoting children's learning. The third more closely relates to the learning plan itself—the instructional design we choose to employ. We'll first look at the fundamental principles that transcend understanding of any mathematical concept, and then address potential learning trajectories that are consistent with and lead to development of the mathematical understandings considered most essential for ensuring children's mathematical success upon entering math programs at the kindergarten through grade 2 level.

Precursors to formal understandings of number concept and operations

The elementary learning expectations set forth in *Principles and Standards for School Mathematics* (NCTM, 2000) place importance on the "Number and Operations" standard (along with "Geometry"). We follow by asking:

1. What can young children do that is sound both mathematically and developmentally?
2. What experiences can we provide to prepare them to meet future expectations?

Focusing on precursors…

- positively contributes to a young child's preparation to meet the elementary "Number and Operations" standard;

- helps to keep activities realistic and developmentally sound; and

- provides means for incrementally increasing complexity of a broad variety of mathematical activities as a child's understanding grows.

In exploring the research that informs these questions, we find that a solution to the first provides insight into the structure of a solution to the second. For example, in the context of early elementary objectives such as simple addition and subtraction, we of course do not often find a child less than four years old who can add or subtract numbers that are several digits removed from one another. However, it is not sufficient for us to say, as many would superficially interpret Piaget (Piaget & Szeminska, 1952), that prior to reaching an operational thinking stage of thinking—around age seven or eight—children cannot possibly begin to understand the operations of addition and subtraction. Rather more informative is that upon close analysis of conceptual shortcomings of school age children who do not understand such operations, we find an absence of certain pre-formal conceptions that are essential to formal understanding of addition and subtraction. Some of these conceptions—connecting one distinct number to each distinct object during counting, for instance—are easily within grasp of preschool children. Determining what conceptual understandings are appropriate for preschool children, when and in what order they are most effectively addressed, and what broader formal understandings they will eventually support, helps us to formulate a reasonable learning trajectory that will better prepare young children for the elementary mathematics standards and curriculum to come. As you review the following precursors, you will note that they are interrelated, and in some manner all apply to each trajectory discussed in the next section.

One-to-one Correspondence: Imagine a young shepherd boy to whom a flock of sheep is entrusted. The boy keeps a pouch on each side. In one pouch, there are a number of pebbles, one for each sheep. Though he perhaps knows no "numerals," he nevertheless counts through the sheep several times each day by moving one pebble for each sheep present into the other pouch. When finished, if there is a pebble left over, he knows he has somehow lost one sheep. If he runs one pebble short, he knows he has accidentally acquired another sheep from a nearby flock. Even in the absence of formal counting, he has exhibited one-to-one correspondence. The concept can range from non-numerical matching (as with the shepherd boy), to formal understanding of specific numerals and written symbols that are attached to each object.

Cardinality: The concept of cardinality refers to the understanding that the number of the last object counted in a set is the total number of objects in that set. For instance, the shepherd boy in the above example exhibits an understanding of cardinality if he realizes that the last pebble indicates the number of sheep in his flock. Again, it may be an informal understanding, or it may be a distinct numeral and/or symbol (e.g., 34 sheep) he attaches to that understanding.

Ordinality: When our shepherd boy realizes that each pebble means more sheep, he is exhibiting ordinality. The concept of ordinality refers to his ability to associate smaller numbers of pebbles with fewer sheep, higher numbers of pebbles with a higher number of sheep, and the order of numbers in between. Understanding of ordinality in its most implicit sense may be as simple as grasping the concept of "more or less." As with our other precursors, it may mean ordering objects in a set by number, ordering sets or objects by size or quantity, and/or distinguishing between a continuum of values on a number line.

Conservation: Assuming that the sheep above like to move around and graze, our shepherd boy exhibits conservation of number when he understands that regardless of their position the number of sheep remains the same. In general terms, children who grasp the principle of conservation of quantity are not fooled by rearranging objects, changing their attributes, pressing them closely together or moving them apart—they know there are still the same number of objects. The principle applies likewise to physical properties such volume, mass, length, and so on. Though some conception of conservation may be grasped on an implicit level, it is usually used in reference to a child's understanding when he/she enters Piaget's operational stage.

Sequencing children’s experiences with number and operations

By reviewing both the elementary expectations for learning and what research tells us about when children are developmentally able to understand certain concepts, we can construct a sequence—or trajectory—of number and operations expectations through the preschool years.

Consider the following early elementary number concept expectations (NCTM, 2000) related to understanding of number quantity. Students will:

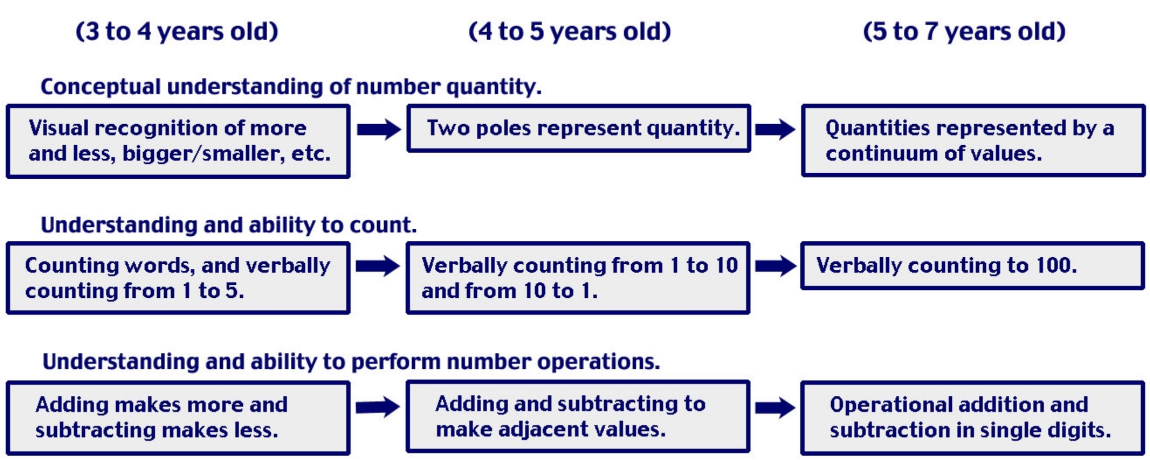
- use multiple models to develop initial understandings of place value and the base ten number system;
- develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections; and
- develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers.

Consider also the early elementary expectations related to counting—that students will:

- count with understanding and recognize "how many" in sets of objects; and
- connect number words and numerals to the quantities they represent, using various physical models and representations.

The trajectories in Figure 1 below each represent a potential preschool continuum of learning expectations. The first two illustrate children’s progress toward the elementary expectations for understanding of number quantity and counting, respectively. The third represents an introductory preschool continuum of learning for addition and subtraction of whole numbers.

Figure 1. Trajectory of preschool children’s understandings of number and operations.



Though research generally indicates that most children are able to reach these levels of understanding at or near the ages indicated, it is noteworthy that individual differences in young children are as pronounced as for the rest of us—the most pronounced consistency is often the lack of consistency. It is also important to expect and to understand the contingencies that exist among continua—children typically must form certain understandings and abilities in one area before they are able to effectively progress to more advanced levels in other areas. For instance, recent research findings have indicated that a child's ability to count is instrumental in supporting a wide range of preschool number conceptions and the ability to perform single-digit addition operations. For more information and research-based strategies for helping young children learn to count, read the article Count With Me! Likewise, the key to a child's progress in ability to perform whole number operations lies in the strategies that he employs. At very young ages, children should begin to progress through increasingly complex cycles for solving simple problems involving operations. Information is provided, along with a separate trajectory toward attainment of elementary standards and expectations, in our article Add With Me! Regardless of the differences that exist among young children, we nevertheless find a constant theme—through attentiveness to sequencing expectations, and by adjusting learning exercises accordingly, we can help promote children's continual growth in understanding by knowing where they should ultimately be heading and by staying at the edge of their level of understanding at any point in time.

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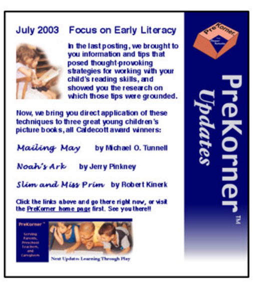

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